

A Study of Inventory Models for Deteriorating Items with Shortages

Prof. (Dr.) Deepak Raj
P.I.E.T. Samalkha (Panipat)

ABSTRACT

Inventory modeling is an important part of Operation Research, which may be used in variety of problems. To make it applicable in real life situations researchers are engaged in modifying the existing models on different parameters under various circumstances. This paper reviewed the inventory models with shortages of different types for deteriorating items with different demand patterns and proposed future need of research in this direction.

Keywords: *Inventory Models, Deterioration, Shortage, Operation Research.*

I. INTRODUCTION

One of the most developed fields of Operations Research is inventory modeling. Inventory has been defined as idle resources that possess economic value by Monks(1987). Usually, it is an important component of the investment portfolio of any production system. Keeping an inventory for future sales or use is very common in business. Retail firms, wholesalers, manufacturing companies and even blood banks generally have a stock of goods on hand. Usually the demand rate is decided by the amount of the stock level. The motivational effect on the people may be caused by the presence of stock at times. Large quantities of goods displayed in markets according to seasons motivate the customers to buy more. If the stock is insufficient the customers may prefer some other brands, as shortages will fetch loss to the producers. The shortage or stock out cost is the penalty incurred for being unable to meet the demand when occurs. It has two wings internal and external shortage. Internal shortage occurs when an order of a group or department within the organization is not filled. External shortages can incur backorder cost, present profit loss and future profit loss. Internal shortages can result in lost production and delay in a completion date. On the other hand, deterioration is an important natural phenomenon and the consequent loss due to decay of items may be quite significant. Mainly when, physical goods are stocked for future use, in some items such as medicines, foodstuff, dairy items, volatile liquids, the process of deterioration is observed. Hence effect of deterioration is very important in many inventory systems.

Research in this direction began with the work of Whittin (1957) who considered fashion goods deteriorating at the end of a prescribed storage period. Ghar and Schrader (1963) developed an inventory model with a constant rate of deterioration. An order level inventory model for items deteriorating at a constant rate was discussed by Shah and Jaiswal (1977). In all these models, the demand rate and the deterioration rate were constant and shortages were not allowed.

An interesting subset of inventory modeling is the mathematical modeling of deteriorating items with shortages. The literature related to deteriorating items with shortages is scattered and no comprehensive up to date discussion of these models is available. This paper presents a complete survey of the published literature in mathematical modeling of deteriorating items with - 52 - shortages and proposed future research directions needed in this field.

Numerous researches have been carried out incorporating time demand patterns into inventory models. The time dependent demand patterns, used in existing models are, mainly,

- linearly time dependent
- exponentially time dependent

The time dependent demand patterns reported above are linear, that is, the demand increases continuously with time or decreases continuously along with the time. Dave and Patel (1981), Dutta and Pal (1992), considered time proportional demand. Goyal (1986) considered linear trend in demand. Hariga and Benkherouf (1994) considered exponential time varying demand for deteriorating items.

Hill(1995) proposed a time dependent demand pattern by considering it as the combination of linearly time dependent and exponentially time dependent of demand in two successive time periods over the entire time horizon and termed as “ramp- type” time dependent demand pattern. Then, inventory models with ramp type demand rate also studied by Mandal and Pal(1998), Wu et al(1999), Wu and Ouyang(2000), Wu(2001), are worth mentioning. In these papers, the determination of the optimal replenishment policy requires the determination of the time point, when the inventory level falls to zero. So the following two cases should be examined:

- This time point occurs before the point, where the demand is stabilized,
- This time point occurs after the point, where the demand is stabilized.

Almost all of the researchers examine only the first case. Deng et al(2006) reconsidered the inventory model of Mandal and Pal(1998) and the models of Wu and Ouyang(2001) and studied it exploring these two cases. Skouri et al(2009) extended the work of Deng et al(2007) by introducing a general ramp type demand and considering Weibull distribution deterioration rate.

The works done by Roy(2008), Sabahno(2009), Mirzazadeh(2010), Gayen and Pal(2009) are some of the models for deteriorating items based on different realistic situations.

Deterioration is defined as decay, spoilage, loss of utility of the product as defined by Shah and Shukla(2009). Product such as vegetables, fish, medicine, blood, radio-active chemicals have finite self life and start to deteriorate once they are produced.

Obsolescence refers to inventories that become obsolete at a certain time, for instance because of rapid changes in technology, or the introduction of a new product by a competitor. If the rate of obsolescence, deterioration or amelioration is not sufficiently low, its impact on modeling of such an inventory system can not be ignored. Moon et al(2005) considered ameliorating/ deteriorating items on an inventory model with time varying demand pattern. The model of Balkhi(2004) and Lo et al.(2007) may also be mentioned.

In the literature survey by Raafat(1991), Shah and Shah(2000), Goyal and Giri(2001), the details can be found for deteriorating items.

Dave(1985) modified the model of Dave and Pandya(1985) with special sales incorporating shortages. Later Dutta and Pal(1991) designed the similar problem with shortages where the deterioration rate of the items is constant and deterioration is assumed to start only after a fixed period of time from the instant of their arrival in stock. Beckherouf (1995) developed an algorithm for solving the problem of determining an optimal replenishment policy for inventories of perishable goods with constant rate of deterioration and with decreasing demand rate over a certain time period.

Manna and Chaudhuri(2003) noted that ramp type demand pattern is generally followed by new brand of consumer goods coming in the market. But for fashionable products as well as for seasonable products, the steady demand will never be continued indefinitely. Rather it would be followed by decrement with respect to time after a period of time and becomes asymptotic in nature. Thus the demand may be illustrated by three successive time periods that classified time dependent ramp type function, viz,

- first phase the demand increases with time
- after that it becomes steady
- towards the end in the final phase it decreases and becomes asymptotic.

Some consumer goods for which stock-dependent demand pattern can be noticed are subject to deterioration. Dutta and Pal(1990) investigated a model assuming the demand rate to be linear function of the on-hand inventory incorporating deterioration effect and allowing shortages which are completely backlogged for both infinite and finite time-horizon. Deb and Choudhuri(1989) were the first to develop the inventory problem with linearly increasing demand to allow shortages. Mainly there are two types of shortages, viz

- inventory followed by shortages (IFS)
- shortages followed by inventory (SFI)

In IFS policy, it is assumed that each of the $(n-1)$ cycles start with replenishment; the inventory is held for a certain period and then shortages are allowed to occur. Shortages are not permitted in the last replenishment. This IFS policy was then studied by Bahari Kasani(1989), Goswami and Choudhuri(1991), Chung and Ting(1993), Hariga(1995), Jalan et al(1996), Giri and Chaudhuri(1997), Chakrabarti and Chaudhuri(1998) and others. They have been devoted to incorporating a time varying demand into their models for deteriorating items under a variety of circumstances.

SFI policy has emerged in recent years. In the SFI policy, each cycle starts with a shortage and ends with zero stock. The SFI policy was first discussed by Goyal et al(1992) who suggested a new replenishment policy in which shortages are allowed in every cycle. Each cycle starts with a shortage that accumulates until a replenishment is made to clear the backlog. They also proved that the system cost in the SFI policy would be less than that in IFS policy. This policy was adopted in the model of Chakrabarti and Chaudhuri(1997) for an inventory of a perishable commodity with linear trend in demand. Recently Jalan and Chaudhuri(1999) proposed an EOQ model for deteriorating items with exponentially declining demand under SFI policy. Wu et al.(2000) have been developed a deterministic inventory model for deteriorating items with time varying demand and shortage under SFI policy.

Samanta and Roy(2004), have developed a continuous production control inventory model for deteriorating items with shortages. It is assumed that the demand rate and production rate are constants and the distribution of the time to deterioration of an item follows the exponential distribution.

Skouri and Konstantaras(2009) have developed an order level inventory model for deteriorating items. The basic assumption of the model is based on time dependent three branches ramp type demand rate. Thereof

seasonable and fashionable products can be described well with this function, as the nature of demand of these products is increasing at the beginning of the season, steady in the mid of the season, and decreasing at the end of the season. In addition a time dependent backlogging and deterioration rate is assumed. The inventory model is studied under two different replenishment policies:

- starting with no shortages
- starting with shortages

In most of the above mentioned papers, the demand during stock-out period is totally backlogged. But in real life situations, there are customers who are willing to wait and receive their order at the end of stock-out period due to good will of the retailer or for some reasons while others are not. In the last few years considerable attention has been paid to inventory models with partial backlogging. The first work in which customer's impatience functions are proposed seems to be that by Abad(1996,2001) Abad derived a pricing and ordering policy for a variable rate of deterioration and partially backlogging. The partially backlogging was assumed to be exponential function of waiting time till the next replenishment. Dye et al(2007) modified this model taking into consideration the backorder cost and lost sale. Shah and Shukla(2009) also developed a deterministic inventory model in which items are subject to constant deterioration and shortages are allowed. The unsatisfied demand is backlogged which is a function of time. It is assumed that the backlogged units are proportional to waiting time. Thus in this paper, an optimal replenishment schedule is derived under the assumption of waiting time backordering when units in an inventory are subject to constant deterioration. Research on models with partial backlogging for deteriorating items continues with Wang(2002), Uthya and Parvathi(2006).

The production schedules for this system were constructed using traditional scheduling strategy in which each cycle starts with replenishments ends with shortages. Dutta and Pal(1991) investigated the effect of inflation and time-value of money on an inventory model with linear time dependent demand rate allowing shortages in their model.

The stock dependent demand rate models are prepared with some researchers. Hou and Lin (2004) developed an inventory model under inflation and time discounting for deteriorating items with stock dependent selling rate. The selling rate is assumed to be a function of the current inventory level and the rate of deterioration is assumed to be constant. A deterministic economic order quantity (EOQ) inventory model taking into account inflation and time value of money developed for deteriorating items with price and stock dependent selling rates by Hou and Lin (2006). Roy and Chaudhuri (2009) established two production inventory models for deteriorating items when the demand rate depends on the instantaneous inventory level. One model is considered without shortage and the other is with shortage.

In modern age, the busy places like supermarket, municipality market or any where else, the storage area is limited. When an attractive price discount for bulk purchase is available or the cost of procuring goods is higher than other inventory related cost or demand of items is very high or there are some problems in frequent procurement, or the management decide to purchase a large amount of items at a time, rented warehouse is needed. Assuming the deterioration in both warehouses, Sarma (1987), extended his earlier model to the case of infinite replenishment rate with shortages. Pakkula and Achary (1992) extended the two-warehouse inventory model for deteriorating items with finite replenishment rate and shortages, taking time as discrete and continuous variable respectively, where the demand rate is constant. In this model order level inventories for perishable items in two warehouses with same deterioration rates in both the warehouses was considered.

The capacity of own warehouse(OW) is fixed according to the cost and service level of the company when it is built. When the capacity of our OW can not store the excess units, they should be stored in a rented warehouse(RW). Sarma (1987) examined the RW in the form of a central warehousing facility. The RW offers better preserving facility and higher cost than the OW resulting in a lower rate of deterioration for goods. Sarma(1987) obtained an optimal replenishment schedule when the demand rate is constant and OW has known capacity.

Singh et al (2009) considered an inventory model for deteriorating item having two warehouses, one is OW of finite dimension, other RW of infinite dimension, under inflation and time value of money. Deterioration rates of items in the two warehouses may be different, which is time dependent, and shortages are allowed. In this model, due to different facilities and storage environment, inventory holding cost is considered different in different warehouses. The demand rate of items is linear with time. Chiao et al(2008) considered a deteriorating inventory model with two storage facilities, partial backordering and quantity discount. Kumar and Singh (2009) developed a two-warehouse inventory model for perishable inventory items with the First-In-First-Out(FIFO) dispatching policy.

They modified the Last-In-First-Out (LIFO) model of Pakkala and Achary's (1992) considering the deterioration rate in OW is time dependent and in RW is constant and the demand rate is time dependent in both the warehouses. Comparing both the models they concluded that FIFO model is less expensive to operate than the LIFO model, if the mixed effect of deterioration and holding cost in RW is less than that of OW.

The best replenishment policy based on the minimum total relevant costs comparing the different results Teng et al (2001) analysed that the relevant cost is convex with the number of replenishments for fluctuating demand.

Teng and Leung(2004) compared among various inventory shortage models on the basis of maximizing profit. They studied the literature and divided them into four types of models mainly as in the figure. After the comparative analysis, model 3 provides the highest profit when profit margin is higher than the unit holding cost. They also considered the case of constant rate of deterioration and concluded that model 3 has the highest profit among those four models. Otherwise model 4 has the highest profit among them.

Jaggi and Verma(2010) proposed a model where rented warehouse is used to store the excess units over the capacity of the own warehouse. The stock is being transferred from RW to OW in a continuous release pattern with per unit transportation cost being factored in. The solution obtained in the model helps to decide on the feasibility of renting a warehouse.

II. DISCUSSION

In modeling of inventory management, deteriorating items with shortages, lot of discussion is made by different researchers. There is a long way to go in this direction. Dutta Choudhury and Dutta(2010) developed a model with stock dependent demand rate and dual storage facility. Deterioration is not considered in their model. From the above discussion it is clear that in real life situations deterioration may occur and hence shortages. Paul et al. (1996) developed a model with two component demand rate allowing shortages. Thus the demand may be stock dependent up to certain time after that it is constant due to some goodwill of the retailer. This model can be considered with deteriorating items.

The necessity of third storage cannot be ignored and emphasis should be given whether the third storage is needed or not in the context of deteriorating items and allowing shortages.

As decay is the natural process, we cannot ignore this vital factor in inventory model in this age of globalization. At the same time due to wide range of business throughout the world shortage will also be a subsequent stage in this modern business.

In the present scenario of world economy both the factors viz, deterioration and stock-out situations are equally important.

Researchers are engaged in designing newer inventory models taking into consideration the different aspect of real life situations for the rapid growth of economy. At this moment along with other factors equal emphasis should be given on the deterioration and shortages.

Inflation and the value of money is also a key factor in the inventory management and as a result in the modern economy. Keeping this points in view, the inventory model can be designed accordingly to face the practical situations in near future.

REFERENCES

- [1]. Abad, P.L. (2001). Optimal price and order-size for a reseller under partial backlogging. *Computers and operation Research*. 28:53-65
- [2]. Balkhi, Z.T. (2004). On the optimality of inventory models with deteriorating items for demand and onhand inventory dependent production rate. *IMA J. Manage.Math*.15:67-86.
- [3]. Chakrabarti, T.; Chaudhuri,K.S. (1997). An EOQ model for deteriorating items with linear trend in demand and shortages in all cycles. *Int. J. of Production Economics*. 49: 205-213
- [4]. Chakrabarti,T.; Chaudhuri, K.S.(1998). An EOQ model for items with weibull distribution deterioration, shortages and trended demand: an extension of Philips model. *Computers and OPS. Res*.25: 649-657
- [5]. Chung, K-J.; Ting, P-S.(1993) A heuristic for replenishment of deteriorating items with a linear trend in demand. *J. Opl. Res.Soc*.44: 1235-1241 Dave,U.(1985). Inventory returns and special sales in the order-level lot-size system. *Cahiers du C.E.R.O.*: 27: 221-227
- [6]. Dave,U.; Patel, U.K.(1981). (t,Si) policy inventory model for deteriorating items with time proportional demand. *J. of Opl.Res.Soc*.32: 137-142
- [7]. Deng, P-S; Yen,C-P; Tung,C-T and Yu,Y-C. (2006). A technical note for the deteriorating inventory model with exponential time varying demand and partial backlogging. *Information and management sciences*. 77(2):101-108
- [8]. Giri, B.C.; Chaudhuri, K.S. (2003). Economic order quantity model with Weibull deterioration distribution, shortage and ramp type demand. *Int. J of Systems Science*.34(4): 237-243
- [9]. Giri,B.C.; Chaudhuri,K.S. (1997). Heuristic models for deteriorating items with shortages and time varying demand and costs. *Int. J.system sci*.28:153-159
- [10]. Goyal, S.K. (1986). A heuristic for replenishment of trend inventories considering shortages. *J.Opl.Res.Soc*. 39:885-887
- [11]. Hill R.M.(1995). Inventory model for increasing demand followed by level demand. *J of Operational Research Society*.46:1250-1259
- [12]. Jalan, A.K.; Giri,B.C.; Choudhuri, K.S.(2003). An EOQ model with Weibull deterioration distribution, shortage and ramp type demand. *Int. J.Systems.Sci*.34(4): 237- 243
- [13]. Manna, S.K.; Choudhuri, K.S.(2003). An EOQ model with ramp type demand rate, time dependent deterioration rate, unit production cost and shortages. *Int. J. Systems Sci*. 34(4): 237-243
- [14]. Paul,K.; Dutta, T.K.; Choudhari, K.S. and Pal, A.K.(1996), An inventory model with two component demand rate and shortages. *J. of the operational research society*.47: 1029-1036

- [15]. Roy, T.; Chaudhuri, K.S. (2009). A Production - inventory model under stock -dependent demand, Weibull distribution deterioration and shortage. *Int. Federation of Operational Res. Soc.* 16(3):325-346
- [16]. Shah, N.H.; Shukla, K.T. (2009). Deteriorating inventory model for waiting time partial backlogging, *Applied Mathematical Sciences.*3(9): 421-428
- [17]. Teng, J.-T.; Leung, C.-K.(2004). A comparison among various inventory shortage model on the basis of maximizing profit. *Information and Management Science.*15(2): 1-10